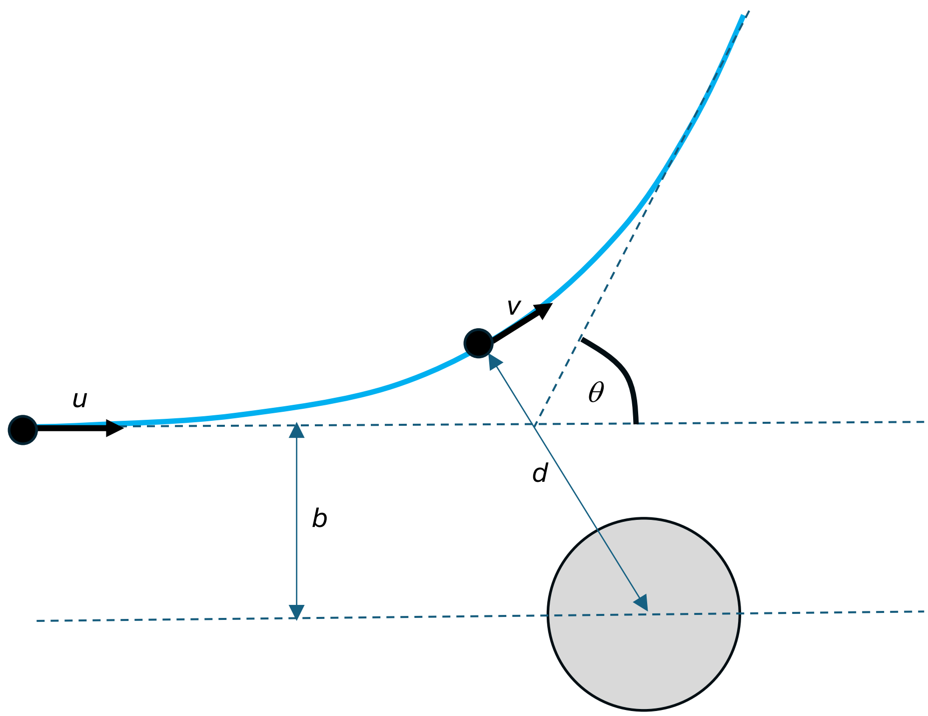
Teacher notes
Topic E

Details of the distance of closest approach in Rutherford scattering

This is a note on the distance of closest approach in Rutherford scattering which is accessible to HL students and makes good use of combining energy and angular momentum conservation. It also makes clear the connection between the impact parameter, the distance of closest approach and the scattering angle.

The figure shows an alpha particle being scattered by a nucleus. The impact parameter *b* is the perpendicular distance between the center of the nucleus and the line through the initial velocity of the alpha particle. The distance of closest approach is *d*. The speed of the alpha particle at the distance of closest approach is *v*. The angle by which the alpha particle is scattered is θ.



Conservation of energy states that:

$$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{k2e \times Ze}{d}$$

or just

$$u^2 = v^2 + \frac{4kZe^2}{md}.$$

Conservation of angular momentum gives

$mub = mvd$ or $u^2b^2 = v^2d^2$.

Substituting we find

$u^2 = \frac{u^2b^2}{d^2} + \frac{4kZe^2}{md}$

or

$d^2u^2 = u^2b^2 + \frac{4kZe^2}{m}d$

or

$$d^2 - \frac{4kZe^2}{mu^2}d - b^2 = 0$$

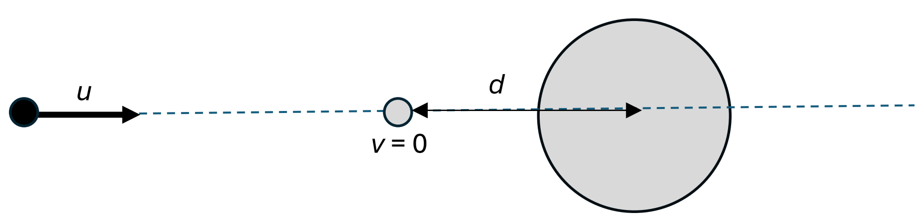
The quadratic equation has solutions

$d = \frac{2kZe^2}{mu^2} \pm \sqrt{\left(\frac{2kZe^2}{mu^2}\right)^2 + b^2}$

We get a positive answer for *d* if we take the plus sign and so finally:

$$d = \frac{2kZe^2}{mu^2} + \sqrt{\left(\frac{2kZe^2}{mu^2}\right)^2 + b^2}$$

If the impact parameter *b* is zero, $d = \frac{4kZe^2}{mu^2}$ which is the standard result for a head on collision:



$$\frac{1}{2}mu^2 = 0 + \frac{k2e \times Ze}{d} \quad \Rightarrow \quad d = \frac{4kZe^2}{mu^2}$$

From $d=\frac{2kZe^2}{mu^2}+\sqrt{\left(\frac{2kZe^2}{mu^2}\right)^2+b^2}$ we see that as the impact parameter gets smaller, the distance of closest approach also gets smaller.

The path of the alpha particle is a hyperbola. With some extra work that involves the geometry of the hyperbola we can also prove that

$$\tan\frac{\theta}{2}=\frac{2kZe^2}{mu^2}\frac{1}{b}$$

This says that as the impact parameter gets smaller the scattering angle increases as we expect. And we have already seen that with *b* smaller the alpha particle gets closer to the nucleus as well.

In summary, as the impact parameter gets smaller the alpha particle gets closer to the nucleus and the scattering angle increases.